

# Charge Fluctuations in a Quantum Dot with a Dissipative Environment

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We consider a multiple tunneling process into a quantum dot capacitively coupled to a dissipative environment. The problem is mapped onto an anisotropic Kondo model in its Coulomb gas representation. The tunneling barrier resistance and the dissipative resistance of the environment correspond to the transverse and the longitudinal Kondo couplings respectively. We thus identify a line in the parameter space of the problem which corresponds to a zero-temperature Berezinskii-Kosterlitz-Thouless like phase transition. The physics of coupling to the environment is elucidated and experimental consequences of the predicted transition are discussed.

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## I. INTRODUCTION

The problem of the Coulomb blockade in zero dimensional quantum dots has been introduced theoretically and studied both theoretically and experimentally over the past decade [1,2]. First studies employed an orthodox model within which the effect of charging has been accounted for classically. More recent studies have included various quantum mechanical aspects of the Coulomb interaction on various levels of rigor. Thus the Altshuler-Aronov zero bias anomaly [3] has been related to the Coulomb blockade [4–7]. In a slightly different language it was also realized that the coupling to a dynamically polarizable environment may modify the system's behavior qualitatively [8,9]. The instantaneous tunneling of an electron into the dot leads (through the Coulomb interaction) to a shake-up excitation of the low-energy modes of the environment. This process, which is very similar in nature to the X-ray edge singularity (see Refs. [8,9] and references therein), results in a power law current-voltage characteristics of a single tunneling barrier connected to a dissipative circuit.

Another remarkable sequence of developments is associated with the study of the quantum fluctuation of the charge on the dot, or multiple tunneling processes [10–16]. The importance of such processes in the vicinity of transmission resonance has been first realized by Glazman and Matveev [11]. Later on Matveev [12] mapped the problem onto the anisotropic Kondo model [17,18]. This mapping is based on a projection of the Hamiltonian onto two charge states of the dot. The two degenerate charge states of the dot (which corresponds, say, to  $N$  and  $N + 1$  electrons respectively), are mapped onto two spin states, “up” and “down”, with the coupling to the leads playing the role of a coupling to a Fermi sea. A small deviation from resonance conditions plays the role of a constant magnetic field in the Kondo problem, leading to Zeeman splitting. Extensions of this problem to other variants have been carried out employing equivalent methods [19].

In the present paper we consider the effect of a dissipative environment on the quantum fluctuations of charge of the dot. This relatively straightforward combination of previously studied effects leads to a qualitatively novel physical situation, which, to the best of our knowledge, has not been discussed before in the present context. The dissipative resistance of the environment,  $r$ , capacitively coupled to the dot, gives rise to a new parameter, important for the physical description of the system. In the language of the anisotropic Kondo model [17,18] this new parameter is precisely equivalent to a (ferromagnetic)  $z$ -axis coupling,  $J_z$ . Let us recall that the resistance of the tunneling barrier,  $r_t$ , is mapped [12] onto the (inverse)  $x$ - $y$  coupling,  $J_\perp$ . As a result one obtains a separatrix line in the parameter space ( $r_t$ - $r$ ) of the system. Crossing this line by varying the values of the respective resistances leads to a zero-temperature phase transition. In the “ferromagnetic” phase the system is overdamped and its behavior is akin to that of a single tunneling barrier connected to a dissipative circuit [8,9]. Namely, the peak conductance decreases as a certain power of temperature. By contrast, in the “anti-ferromagnetic” phase, the peak height undergoes re-entrance behavior and eventually approaches a perfect conductance limit [12] (for a single channel tunneling contact).

Technically we proceed by mapping the problem of multiple tunneling into the dot, onto that of a classical neutral gas in one dimension with long range interactions. In the past, such a mapping has been employed in the context of the anisotropic Kondo problem [17,18] and dissipative quantum tunneling [20]. In the latter context the analogous phase transition corresponding to localization-delocalization in double quantum well was exploited extensively. The quantum dot realization of this transition, proposed here, may be the most suitable for experimental realization. A related, though qualitatively different gas, which also can be realized in the present context, has been employed in the study of a one-dimensional Luttinger model with an impurity [21]. Here we restrict ourselves to a study of a quantum dot

connected to a lead through a single tunnel barrier, occasionally referred to in the literature as a “single electron box”. We discuss the mapping onto a classical gas and the subsequent renormalization group (RG) procedure on the level of the partition function of such a “box”. A quantity more directly related to the experiment is the conductance through a dot with the two barriers [15,16]. We believe, though, that our calculations reflect the experimental situation with  $r_t$  being the resistance of the weakest of the two barriers. We also require that our dot is large enough, in a sense that one may disregard the discreteness of its single particle spectrum.

The outline of this paper is the following. In Section II we define our problem in terms of a model Hamiltonian and an equivalent electrical circuit, and discuss qualitatively the effect of the environment. We also discuss a complete solution of the problem in lowest order in the tunneling amplitude. Mapping the quantum fluctuations of the charge onto a classical Coulomb gas is presented in Section III. The fully and partially screened cases are solved using a renormalization group procedure. In Section IV we discuss the physics of our results and briefly comment on possible generalizations. Technical details of the derivation are presented in two Appendices.

## II. MODEL HAMILTONIAN AND AN EQUIVALENT CIRCUIT

We consider a small capacitance dot, described by the Hamiltonian

$$\mathcal{H}_0 = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \frac{V^{[0]}}{2} \left( \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} - \mathcal{N}_0 \right)^2, \quad (1)$$

where  $a_{\alpha}^{\dagger} (a_{\alpha})$  is a creation (annihilation) operator of an electron in an exact single particle state  $\alpha$  (the latter is defined including disorder potential and spin);  $V^{[0]}/2 = e^2/2C$  is the charging energy, associated with the total charge on the dot ( $C$  is the dot’s self-capacitance);  $\mathcal{N}_0$  is the effective charge of the positive background. This may be regarded as the large wave-length ( $q=0$ ) component of the interaction. Other,  $q \neq 0$ , components are ignored. The dot is assumed to be weakly coupled to a conducting lead, described by a gas of non-interacting electrons

$$\mathcal{H}_{lead} = \sum_k \epsilon_k d_k^{\dagger} d_k, \quad (2)$$

where  $d_k, d_k^{\dagger}$  are Fermi operators. The coupling to the lead is given by

$$\mathcal{H}_{coupling} = \sum_{k,\alpha} (W_{k\alpha} d_k^{\dagger} a_{\alpha} + W_{k\alpha}^* a_{\alpha}^{\dagger} d_k). \quad (3)$$

Below we shall consider a single point contact and impose  $W_{k\alpha} = W$  [22]. The parameter  $W$  is related to

the bare (high temperature) tunneling resistance,  $R_t = (2\pi\hbar/e^2)r_t$ , where

$$(r_t)^{-1} = (2\pi)^2 |W|^2 \nu^{[0]} \nu^{[l]}, \quad (4)$$

where  $\nu^{[0]}$  and  $\nu^{[l]}$  are the (bare) density of states (DOS) of the dot and the lead respectively. Finally, we shall assume that the dot is embedded in an electromagnetic environment, felt by the dot as an external noise. This noisy background is described as an effective time dependent term in the Hamiltonian

$$\mathcal{H}_{noise} = \eta(t) \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}. \quad (5)$$

Below we specify the noise,  $\eta(t)$ , in terms of the environment’s characteristics. Equivalently, we could say that the dot is connected to a bath of harmonic oscillators which are integrated out. The spectral density of  $\eta(t)$  reflects the nature of the environment [20]. In this sense the averaging over noise should be understood as annealed and not as quenched. The total effective Hamiltonian of the system is thus

$$\mathcal{H} = \mathcal{H}_{dot} + \mathcal{H}_{lead} + \mathcal{H}_{coupling} + \mathcal{H}_{noise}. \quad (6)$$

The physical effect of the environment [8,9] can be understood in the following manner. Imagine that an electron has been injected into the dot. The total effective charge,  $e(\langle N \rangle - \mathcal{N}_0)$ , interacts with charges in the environment, leading to their redistribution (polarization). The polarized charge of the environment reduces the energy cost of adding (or removing) an electron to (or from) the dot. There is a certain finite time scale characterizing this redistribution of the environmental charge, rendering the effective interaction in the dot non-instantaneous (retarded) [23,6]. This time constant can be described as an RC time of an effective electric circuit. It might appear that the results obtained through the ensuing analysis are highly non-universal and depend on the particular choice of the model for the environment. We stress that the effect of the environment is quite general, the only model dependent feature being the concrete form of the screened zero-mode interaction,  $V(\omega)$  (see below). It enters through the dependence of  $V(\omega)$  on an effective impedance of the environment [24], and is due to the fact that details of the environment’s polarization may depend on various objects located far from the dot.

Fig. 1 shows a simplified equivalent circuit depicting an experiment of an electron injection into a quantum dot. The dot is assumed to be weakly coupled to the lead;  $C$  represents the self-capacitance of the dot. The dot is capacitively coupled (through  $C_G$ ) to a dissipative environment whose impedance is  $Z(\omega)$ . The source  $U$  represents the equilibrium voltage noise of the environment.

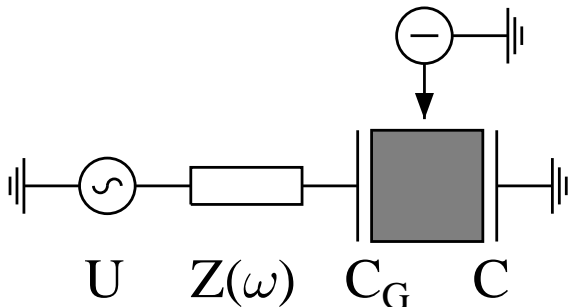


FIG. 1. An equivalent circuit describing an experimental setup of an electron injection into a dot. For details see the text.

In the lowest order in the tunneling amplitude,  $|W|^2$ , the conductance of the tunneling junction is fully determined by the DOS of the dot. The latter may be obtained through the imaginary-time one particle Green function of the Hamiltonian  $\mathcal{H}_0 + \mathcal{H}_{noise}$ . It is, of course, strongly modified by the effects of interaction and coupling to the environment. For completeness we briefly reproduce the calculations of Ref. [7] in Appendix A, which lead to the following expression for the one particle Green function

$$\mathcal{G}_\alpha(\tau) = \mathcal{G}_\alpha^{[0]}(\tau) e^{-S(\tau)}. \quad (7)$$

Here the effective action,  $S$ , is given by

$$S(\tau) = T \sum_{m \neq 0} \frac{V(\omega_m)}{\omega_m^2} (1 - e^{i\omega_m \tau}), \quad (8)$$

where  $T$  is the temperature and  $\omega_m$  are the bosonic Matsubara frequencies,  $\omega_m \equiv 2\pi mT$ . The form of the screened interaction,  $V(\omega_m)$ , can be easily deduced relying on the equivalent circuit picture, Fig. 1. It is shown in Appendix A that

$$V(\omega_m) = V^{(0)} - K(\omega_m), \quad (9)$$

where  $K$  is the correlator of the Gaussian noise felt by a dot. In the imaginary time domain  $K(\tau - \tau') = \langle \eta(\tau) \eta(\tau') \rangle$ , and may be evaluated employing the fluctuation-dissipation theorem. Consulting Fig. 1 the correlator of the total noise is in the equivalent circuit (in terms of real frequencies) is:  $\langle UU \rangle_\omega = e^2 i\omega Z_{tot}(\omega)$ , where the total impedance of the equivalent circuit is  $Z_{tot} = (i\omega C)^{-1} + (i\omega C_G)^{-1} + Z(\omega)$ . The corresponding voltage drop on the dot is  $\eta = U(i\omega C)^{-1}/Z_{tot}$ . Substituting the above given expressions in Eq. (9) and rewriting it in a finite temperature form one obtains

$$V(\omega_m) = \frac{e^2}{C} \frac{C/C_G + |\omega_m| ZC}{1 + C/C_G + |\omega_m| ZC}. \quad (10)$$

In the high frequency limit the interaction is unscreened, whereas in the low frequency limit we obtain a (partially)

screened interaction, where  $C$  is replaced by  $C + C_G$ . The scale of the crossover frequency is given by  $\Omega^{-1} = |Z(\Omega)|C$  [25].

Coming back to the action, Eq. (8), the long time behavior of  $S(\tau)$  is given by the small frequency limit of  $V(\omega_m)$ , yielding

$$S(\tau) \approx |\tau| \frac{e^2}{2(C + C_G)} \text{ for } ZC_G \ll \tau \ll \beta. \quad (11)$$

This is the case of a partially screened interaction,  $C \rightarrow (C + C_G)$ . One expects that in this case the addition of charge to the dot costs finite energy even at long times, which is why Eq. (11) is linear in  $\tau$ . This immediately implies the existence of a gap in the DOS, which is a manifestation of the Coulomb blockade. In the special case of a fully screened interaction ( $C_G \rightarrow \infty$ ), the long time interaction vanishes, i.e. we expect  $V(\omega_m \rightarrow 0) = 0$ . This is the case of a single tunneling contact in series with a classical impedance,  $Z$ , considered in Refs. [8,9]. For a purely Ohmic environment,  $Z = R$ , the interaction is

$$V(\omega_m) = V^{[0]} \frac{|\omega_m|}{\Omega + |\omega_m|}, \quad (12)$$

with  $\Omega \equiv (RC)^{-1}$ . On short time scales ( $\omega > \Omega$ ) before the environment becomes fully polarized to screen out the extra charge injected into the dot, the charging energy is finite. Thus the injection of an electron can be considered as “tunneling under the barrier” in the time direction. The very same physics has been discussed in the context of tunneling into two-dimensional systems with diffusive disorder [23,6]. The related energy cost on short time scales suppresses free particle exchange between the dot and the particle reservoir (although Coulomb blockade in its strict sense is absent). This leads to a suppression of the DOS, hence to a zero bias anomaly. Substituting Eq. (12) in Eq. (8) one obtains for  $\Omega^{-1} \ll \tau \ll \beta$

$$S(\tau) \approx 2r \ln(\Omega|\tau|), \quad (13)$$

where

$$r \equiv \frac{V^{(0)}}{2\pi\Omega} = \frac{R}{(2\pi\hbar/e^2)} \quad (14)$$

is the dimensionless resistance of the environment. This form of  $S$  leads to a power law DOS [7] and hence power law current-voltage characteristics [8,9].

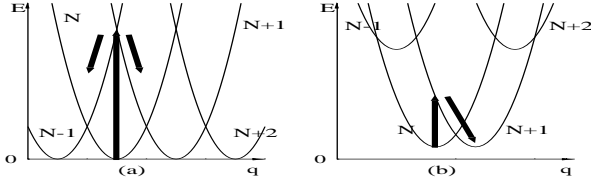


FIG. 2. Energy of the dot with  $N$  particles as function of the gate capacitance charge,  $q$ . Instantaneous tunneling of an electron into the dot, and subsequent slow relaxation of the charge are indicated by arrows. (a) Full screening,  $C_G = \infty$ . (b) Partial screening, a resonance scenario.

Before concluding this section we draw a qualitative picture of the difference between the fully and partially screened interaction scenarios. Consider first the case of  $C_G \rightarrow \infty$  and denote the number of electrons on the dot by  $N$ . At long times the environment polarizes, such that the total neutralizing positive charge (including both the original positive background on the dot,  $\mathcal{N}_0$ , and the accumulated charge on the gate capacitance,  $q$ ) exactly balance off the charge  $N$ . Fig. 2a depicts the charging energy of the dot as function of  $q$ , for various values of  $N$ . This is given by  $E_N(q) = e^2(N - \mathcal{N}_0 - q)^2/2C$ . The fact that the center of each parabola corresponds to  $E_N = 0$  is a manifestation of perfect screening. Upon adding an electron to the dot ( $N \rightarrow N + 1$ ), the charging energy is initially increased, but at later times  $q$  relaxes towards the optimal value corresponding to  $N + 1$ . A

very different situation arises for a partially screened interaction,  $C_G < \infty$ . The charging energy is now given by  $E_N(q) = e^2(N - \mathcal{N}_0 - q)^2/2C + e^2q^2/2C_G$ . The curvature of each parabola corresponds to  $e^2/2C + e^2/2C_G$ , whereas the centers of the various parabola are located on the parabola  $e^2/2(C + C_G)$ . Adding an electron to the system initially costs some energy (as in the previous case), but eventually the system relaxes to the partially screened state whose energy, although less than in the first moment after tunneling, is not in general equal to that of before tunneling event. By tuning  $\mathcal{N}_0$  it is possible to have at most two states having the same final energy. Exactly this resonance scenario (with  $\mathcal{N}_0 = N + \frac{1}{2}$ ) is depicted in Fig. 2b. This picture will be useful while considering multiple tunneling events, or quantum fluctuations of charge on the dot.

### III. QUANTUM FLUCTUATIONS OF CHARGE

Our aim now is to address the full Hamiltonian, Eq. (6), including the possibility of multiple tunneling events. To simplify calculations we first consider a partition function of the system. We formally exactly expand it in powers of  $\mathcal{H}_{coupling}$  and calculate separately each term of the expansion. Details of this procedure may be found in Appendix B. As a result one obtains the normalized partition function of the dot coupled to the lead in the following form

$$\frac{Z}{Z^{[0]}Z^{[l]}} = \sum_{M=0}^{\infty} \frac{\theta^{2M}}{M!M!} \int_0^{\beta} \dots \int_0^{\beta} d(\Omega\tau_1) \dots d(\Omega\tau_{2M}) \exp \left\{ \sum_{i < j}^{2M} (-1)^{i+j} [2 \ln(\Omega|\tau_i - \tau_j|) + S(\tau_i - \tau_j)] \right\}, \quad (15)$$

where  $Z^{[0]}$  and  $Z^{[l]}$  are partition functions of uncoupled dot and lead correspondingly,  $S(\tau)$  is defined by Eq. (8) and

$$\theta \equiv \sqrt{|W|^2 \nu^{[0]} \nu^{[l]}} = \frac{1}{2\pi} \sqrt{\frac{1}{r_t}}. \quad (16)$$

The r.h.s. of Eq. (15) may be considered as the grand-canonical partition function of a *classical* one-dimensional *neutral* gas consisting of positive and negative charges which interact through the potential  $2 \ln \Omega|\tau| + S(\tau)$ . The quantity  $\theta$ , related to the resistance of the tunneling barrier by Eq. (16) (cf. Eq. (4)), represents the *fugacity* of the gas. The interaction potential of the neutral gas deserves some elaboration. The first term,  $2 \ln \Omega|\tau|$ , came from the Green functions of non-interacting electrons in the dot and the lead. At finite temperature one should substitute  $\tau \rightarrow \sin(\pi\tau/\beta)\beta/\pi$ . One should also understand that the interaction potential given above is for large separation between charges ( $\Omega\tau \gg 1$ ), for  $\tau \rightarrow 0$  it should vanish. Note that  $S(\tau)$

obeys this property as well as periodicity with  $\beta$  (by definition, cf. Eq. (8)).

Before treating specific examples we make a few comments. Our gas which consists of the same number of positive and negative charges is fundamentally different from the logarithmically interacting gas known from the theory of random matrices [26]. The latter contains charges of the same sign placed in a confining potential. In our case the positions of the charges along the one dimensional (imaginary) time axis correspond to the times of the (instantaneous) tunneling events: positive charges – tunneling into the dot; negative – tunneling out of the dot. In general, any ordering of the charges, compatible with global neutrality is permissible. Neutrality follows from the fact that the partition function is given by a trace, thus forcing the initial and final charge states to coincide. At finite temperatures the interaction becomes periodic over the period  $0 \leq \tau < \beta$ . The resulting gas is thus defined on a ring of circumference  $\beta$ .

### A. Fully Screened Interaction

As has been discussed in Section II, the fully screened interaction scenario corresponds to an infinite gate capacitance,  $C_G \rightarrow \infty$ , in the equivalent circuit (cf. Fig. 1). This situation is equivalent to the case of a circuit with a single tunneling barrier, considered in Refs. [8,9]

$$\frac{Z}{Z^{[0]}Z^{[l]}} = \sum_{M=0}^{\infty} \frac{\theta^{2M}}{M!M!} \int_0^{\beta} \dots \int_0^{\beta} d(\Omega\tau_1) \dots d(\Omega\tau_{2M}) \exp \left\{ (2+2r) \sum_{i<j}^{2M} (-1)^{i+j} \ln(\Omega|\tau_i - \tau_j|) \right\}. \quad (17)$$

This form breaks down when inter-charge distances are small, i.e. when the density of the Coulomb gas is high. This is a partition function of a classical 1d gas of unit charges interacting through a 2d Coulomb (logarithmic) potential at temperature  $(2+2r)^{-1}$  and fugacity  $\theta$ . We denote this gas as *non-alternating*, meaning that *any* neutral sequence of charges is allowed. The correspondence between the quantum dot problem and the classical Coulomb gas is summarized in a Table 1. Mapping onto the same problem has been found in the context of dissipative tunneling in a periodic potential [20] and more recently in the study of the Luttinger liquid with an impurity [21]. The analogy stems from the observation that the Coulomb gas, Eq. (17), may be obtained from the partition function of a one-dimensional quantum bosonic model with an action

$$S[\varphi(x, \tau)] = \frac{1}{2+2r} \int dx d\tau \varphi (\partial_{\tau}^2 + \partial_x^2) \varphi - 2\theta\Omega \int d\tau \cos(\sqrt{4\pi}\varphi(x=0, \tau)). \quad (18)$$

After integrating the bosonic field at all points except  $x=0$ , one ends up with the effective action of the form

$$S[\varphi(\tau)] = \int d\tau \left[ \frac{1}{1+r} \varphi |\partial_{\tau}| \varphi - 2\theta\Omega \cos(\sqrt{4\pi}\varphi) \right]. \quad (19)$$

Exponentiating the action and expanding in powers of  $\theta$  one directly arrives at Eq. (17) where  $\Omega$  plays the role of the high energy cutoff.

We follow here an approach based on a renormalization group (RG) study of the model [27,21]. One integrates out the high energy (short time) degrees of freedom by continuously rescaling  $\Omega \rightarrow \Omega/\xi$ , where  $\xi$  varies from 1 up to  $\Omega/T$  ( $\beta = T^{-1}$  is the maximal possible size of the classical gas). As a result one arrives at the following well-known RG equations [27,21]

$$\frac{d\theta}{d \ln \xi} = -r\theta, \quad (20a)$$

$$\frac{dr}{d \ln \xi} = 0. \quad (20b)$$

in lowest order in tunneling. We provide here a systematic treatment of higher order processes. Below we shall consider a purely Ohmic environment. The effective action to be substituted in the expression for the partition function is given (for  $(\Omega|\tau_i - \tau_j| \gg 1)$  by Eq. (13). Then the partition function is

The resulting RG trajectories on the  $(r, \theta)$  plane are depicted on Fig. 3a. In the context of a Luttinger liquid with an impurity,  $r$  represents the strength of the interaction and is therefore a feature of the bulk system. The fugacity  $\theta$  represents the strength of the point-like impurity. It is then clear that, while the interaction in the liquid may renormalize the impurity strength, the local impurity cannot renormalize the interaction strength in the bulk [28]. The separatrix  $r=0$  corresponds to the presence of a Berezinskii–Kosterlitz–Thouless (BKT) like phase transition in a system of logarithmically interacting particles [29]. The line  $r=0$  corresponds to the non-interacting system (cf. Eq. (15) with  $S=0$ ); evidently the tunneling transparency is not renormalized on this line.

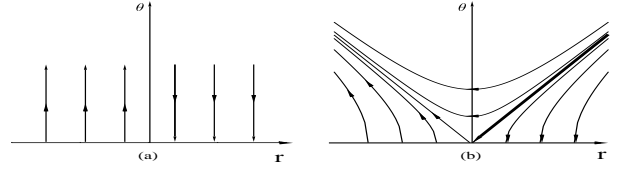


FIG. 3. RG flows of the Coulomb gas: (a) a non-alternating gas; (b) an alternating gas.

For positive  $r$  (which is the only physically meaningful scenario in the present context) the fugacity  $\theta \sim |W|$  is renormalized down,  $\theta \sim \xi^{-r}$ . This provides a justification for a lowest order (in  $W$ ) perturbation theory at low temperatures, which was employed in Section II. Indeed, integrating Eq. (20a) and assuming that the relation between fugacity and resistance, Eq. (16), is conserved in the process of renormalization, one obtains

$$r_t \sim T^{-2r}. \quad (21)$$

This zero-bias anomaly behavior has been derived previously [8,9,7] in a calculation which was non-perturbative in interaction, but perturbative in the tunneling. A similar power-law divergence of the resistance takes place in a repulsive Luttinger liquid with a single impurity [21].

## B. Partially Screened Interaction

We now turn to a somewhat less trivial situation, that of a partially screened interaction. This corresponds to a finite  $C_G$  in Fig. 1. The ground state energies corresponding to different numbers of electrons in the dot are generically not degenerate. We refer though to a particular scenario where the gate voltage (effectively  $\mathcal{N}_0$  in Eq.(1)) is tuned such that the ground state energies corresponding to the charging states  $N$ ,  $N + 1$  are degenerate, cf. Fig. 2b. Injecting an electron to form any other charge state costs at least energy equal to  $\tilde{\Omega} = e^2/2(C + C_G)$  at long time scale. If we set the temperature to be less than this value one may neglect such states. Consequently the charge on the dot can

fluctuate only between  $N$  and  $N + 1$ . The corresponding classical model is that of a gas of strictly alternating charges: a positive charge (tunneling into the dot) is followed by a negative charge (tunneling out of the dot). Similarly to the fully screened scenario discussed above, and following the same lines of derivation, the fact that the (two) available states are degenerate renders the long time ( $\tau \gg \tilde{\Omega}^{-1}$ ) interaction logarithmic. The coefficient in front of the logarithm is determined by the slope of  $V(\omega)$  at small  $\omega$ , which is given according to Eq. (10) by

$$\tilde{r} \equiv r \left[ \frac{C_G}{C + C_G} \right]^2. \quad (22)$$

As a result, the normalized partition function takes the form (see Appendix B)

$$\frac{Z}{Z^{[0]}Z^{[l]}} = \sum_{M=0}^{\infty} \theta^{2M} \int_0^{\beta} d(\tilde{\Omega}\tau_1) \int_0^{\tau_1} d(\tilde{\Omega}\tau_2) \dots \int_0^{\tau_{2M-1}} d(\tilde{\Omega}\tau_{2M}) \exp \left\{ (2 + 2\tilde{r}) \sum_{i < j}^{2M} (-1)^{i+j} \ln(\tilde{\Omega}|\tau_i - \tau_j|) \right\}. \quad (23)$$

The very same Coulomb gas was first discovered by Yuval and Anderson in the context of the Kondo problem [17]. In that case alternating positive and negative charges correspond to the up and down spin flips. The mapping between an isolated dot near the Coulomb resonance and the anisotropic Kondo problem was first presented in Ref. [12]. There the coupling to the environment was absent ( $\tilde{r} = 0$ ) and the inverse temperature of the gas was exactly 2 (see Table 1), rendering the corresponding Kondo problem to be always anti-ferromagnetic. Here we have introduced a coupling to the dissipative environment, which may change the situation qualitatively by driving the system through anti-ferromagnetic-ferromagnetic transition. Note that a small deviation from exact resonance conditions plays the role of Zeeman splitting due to a constant magnetic field in the Kondo model. In the case of the Kondo problem both the fugacity and the inverse temperature are properties of the local spin, hence both may be renormalized. The RG equations assume the form [18,29]

$$\frac{d\theta}{d\ln\xi} = -\tilde{r}\theta; \quad (24a)$$

$$\frac{d\tilde{r}}{d\ln\xi} = -4\theta^2. \quad (24b)$$

In the limit of extremely small fugacity,  $\theta \ll |\tilde{r}|$ , the gas is so dilute that the requirement of alternation does not play any role. Indeed in this case the RG equations of alternating gas coincide with Eqs. (20a) and (20b). To handle the case of finite fugacity one notices that

$$\tilde{r}^2 - (2\theta)^2 = \text{const}. \quad (25)$$

As a result the RG trajectories are hyperbola in the  $(\tilde{r}, \theta)$  plane. The RG diagram is depicted in Fig. 3b. The most

striking feature of this diagram is the presence of the separatrix at  $\tilde{r} = 2\theta$ . In terms of the bare parameters of the problem the equation for the separatrix has the form

$$\pi\sqrt{r_t}\tilde{r} = 1. \quad (26)$$

The low temperature behavior of the system is qualitatively different, depending on whether the bare parameters are chosen to be below or above the separatrix line.

Let us first consider the system below the separatrix,  $r_t < (\pi\tilde{r})^{-2}$ . In this case the fugacity scales down, and the resulting Coulomb gas is dilute (the system is in the “cold” phase, i.e., overdamped by the environmental resistance). Lowest order perturbation theory should be adequate while renormalization provides some corrections. One can easily integrate Eqs. (24a) and (24b) by putting

$$\tilde{r}(\xi) = A \coth(A \ln \xi + B); \quad 2\theta(\xi) = A / \sinh(A \ln \xi + B), \quad (27)$$

where  $A^2 \equiv \tilde{r}^2 - (2\theta)^2$  and  $\coth B \equiv \tilde{r}/A$ . For  $T \ll \tilde{\Omega}$  one obtains

$$r_t(T) \sim T^{-2A}. \quad (28)$$

If  $\theta \ll \tilde{r}$ , then  $A \approx \tilde{r}$  and Eq. (28) coincides with the result of perturbation theory (in the tunneling strength), cf. also Eq. (21). In the more general case the low temperature exponent is somewhat smaller than the naive perturbation theory prediction, indeed  $A < \tilde{r}$ .

At the separatrix line, Eq. (26), the system undergoes a zero-temperature BKT phase transition. Above the

separatrix line,  $r_t > (\pi\tilde{r})^{-2}$ , the fugacity exhibits reentrance behavior and eventually is renormalized up. The corresponding Coulomb gas is in the “hot” or plasma phase. Note that the dot without a dissipative environment ( $\tilde{r} = 0$ ) is always in the hot phase. This latter case has been extensively studied since the work of Matveev [12] (see e.g. Ref. [13]). The temperature below which the reentrance behavior takes place is analogous to the Kondo temperature,  $T_K$ . To find  $T_K$  we integrate Eqs. (24a) and (24b) by the substitution

$$\tilde{r}(\xi) = -\tilde{A} \tan(\tilde{A} \ln \xi - \tilde{B}); \quad 2\theta(\xi) = \tilde{A} / \cos(\tilde{A} \ln \xi - \tilde{B}), \quad (29)$$

where  $\tilde{A}^2 \equiv (2\theta)^2 - \tilde{r}^2$  and  $\tan \tilde{B} \equiv \tilde{r}/\tilde{A}$ . The Kondo temperature may be estimated by equating the argument of the tan to  $\pi/2$ . This way one obtains

$$T_K = \tilde{\Omega} \exp \left\{ - \left( \frac{\pi}{2} + \tilde{B} \right) / \tilde{A} \right\}. \quad (30)$$

The Kondo temperature is maximal for an isolated dot,  $\tilde{r} = 0$ . In this case Eq. (30) reduces to [13]  $T_K = V^{[0]} \exp\{-\pi^2 \sqrt{r_t}/2\}$ . Thus even in the best case the Kondo temperature is exponentially small in the root of the tunneling resistance. In order to assure the validity of this theory the inequality  $T_K > \Delta$  should be satisfied, otherwise the assumption about a continuous spectrum (“metallic dot”) is not valid and one has to deal with the discrete spectrum limit. For a finite value of the environmental resistance,  $\tilde{r}$ , the Kondo temperature rapidly approaches zero once the parameters of the system come closer to the separatrix line, Eq. (26). This makes the experimental observation of the reentrance behavior quite difficult in practice. In contrast, the “cold” phase behavior described above seems to be relatively easy to observe by measuring the temperature dependence of the heights of the Coulomb blockade peaks.

#### IV. SUMMARY AND DISCUSSION

We have addressed here the physics of a (quasi)zero-dimensional quantum dot weakly coupled to an ideal lead and capacitively (and not necessarily weakly) coupled to an external dissipative environment. The fact that charges on the dot interact with the environment which has its own dynamics is translated into an effective time dependent potential felt on the dot. In other words, when a charge is added to the dot it polarizes the environment. This process is not instantaneous — it is characterized by an effective RC time. The delayed response of the environment leads to a non-trivial effective time dependent interaction term. The (weak) coupling of the lead to the dot means that charges can tunnel in and out of the dot, resulting in quantum fluctuations of the charge

on the dot. We have considered a single channel coupling. For this to be the case we require that the linear dimension of the opening connecting the dot to the lead does not exceed the Fermi wave length. The methods we employed (mapping onto the Coulomb gas and perturbative RG) are restricted to the case of relatively weak tunneling coupling, that is barrier resistance larger than  $2\pi\hbar/e^2$ . Although we did not restrict ourselves to any finite order in the tunneling coupling, a consistent treatment of the strong coupling fixed point is outside the scope of this article. The nature of an appropriate fixed point was investigated in Ref. [30]. Our main message here is rather the existence of a separatrix in the parameter (resistance) space of the problem, which separates the basins of attraction of the weak and the strong coupled fixed points.

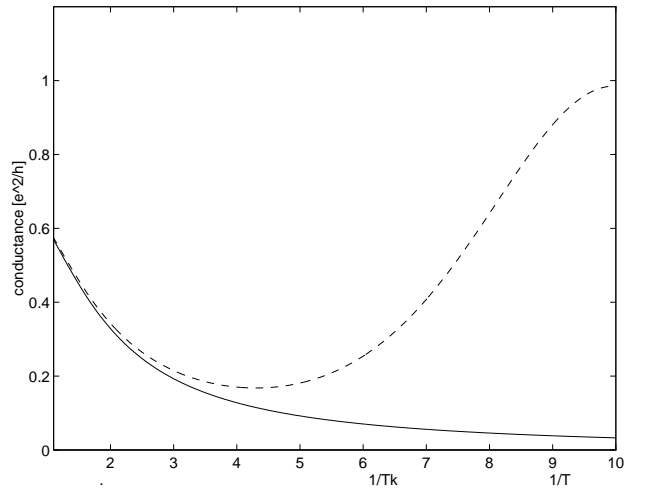


FIG. 4. Schematic behavior of the conductance (in units  $e^2/h$ ) as function of inverse temperature (in units  $1/\Omega$ ) for the overdamped (full line) and underdamped (dashed line) scenario;  $\tilde{r} = .75$ .

We have proceeded by mapping our quantum model onto a classical one-dimensional neutral Coulomb gas. The nature of this classical gas depends crucially on whether the Coulomb term is fully or partially screened. In either case we obtain the corresponding renormalization group flows. In the former case (corresponding to the single tunnel barrier connected to a dissipative circuit [8,11]) the flow is towards the “cold” phase (weak coupling fixed line; Fig. 3a). The results of the perturbative treatment [8,11] are naturally reproduced in this case. That is, at low temperatures the tunneling coupling is suppressed as a certain (positive) anomalous power of temperature (cf. Eq. (21)). The case of the dot with a finite residual capacitance (partial screening) is more intricate. In particular, we found, in agreement with Matveev [12] and others [13] that for sufficiently small dissipation of the environment the problem is equivalent to a anti-ferromagnetic Kondo. The RG flow is towards the strong

coupling, characterizing by the ideal transparency of the contact. If, however, the resistance of the environment exceeds a certain critical value (cf. Eq. (26)) determined by the bare tunneling resistance, the situation changes dramatically. The system turns to be overdamped and its low temperature behavior is determined by a weak coupling fixed line, similar to those of the fully screened situation (with a slightly modified exponent).

The physics of this damping of charge fluctuation by the environment is simple. Consider a dot which is tuned by the gate electrode into a resonance, i.e., the cost (in free energy) of removing one electron from the leads and adding it to the dot is nearly zero. (We presently assume no voltage difference between the two leads connected to the dot). The environmental resistance produces noisy voltage on the gate electrode. Thus, at any given moment, the dot is slightly driven out of resonance. Not surprisingly, charge fluctuations are suppressed by this (time-dependent) deviation from the exact resonance condition. What is more surprising is an existence of a sharp transition at zero temperature (in fact at  $T \leq T_K$ ) between two regimes as a function of the environmental resistance. At a finite temperature one has a smooth crossover between these two limits. The schematic temperature behavior of the tunneling conductance for the two cases distinguished by the value of an environmental dissipation is depicted on Fig. 4. To be within a scenario

of an alternating Coulomb gas the initial temperature should be less than both  $e^2/2(C + C_G)$  and  $(RC)^{-1}$ .

Throughout the above analysis it has been assumed that the relation between the resistance and the fugacity, Eq. (16), remains valid under the renormalization group transformation. It is possible, though, to establish a direct relation between the Coulomb gas description and the conductance. We first note that the current operator,  $\hat{J}$ , is given by the time derivative of the number operator  $\hat{N} = \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$

$$\hat{J} = \frac{i}{\hbar} [\mathcal{H}, \hat{N}] = \Im(W \sum_k d_k^{\dagger} \sum_{\alpha} a_{\alpha}) \equiv \Im \hat{W}. \quad (31)$$

The linear conductance may be expressed, employing the Kubo formula, through current-current correlation function. Evidently, the latter is expressible through the two following correlation functions

$$F(\tau_1 - \tau_2) \equiv \langle \mathcal{T} \hat{W}(\tau_1) \hat{W}^{\dagger}(\tau_2) \rangle, \quad (32a)$$

$$\tilde{F}(\tau_1 - \tau_2) \equiv \langle \mathcal{T} \hat{W}(\tau_1) \hat{W}(\tau_2) \rangle, \quad (32b)$$

where  $\mathcal{T}$  denotes an (imaginary) time ordering. Both  $F$  and  $\tilde{F}$  may be naturally interpreted in a Coulomb gas language. Following the same procedure depicted in Appendix B we arrive at the following expression for e.g.  $F(\tau_1 - \tau_2)$

$$F(\tau_1 - \tau_2) = \frac{1}{Z} \sum_{M=0}^{\infty} \frac{\theta^{2M+2}}{M!M!} \int_0^{\beta} \dots \int_0^{\beta} d(\Omega\tau_3) \dots d(\Omega\tau_{2M+2}) \exp \left\{ \sum_{i < j}^{2M+2} (-1)^{i+j} [2 \ln(\Omega|\tau_i - \tau_j|) + S(\tau_i - \tau_j)] \right\}. \quad (33)$$

This is the partition function of a one-dimensional classical neutral gas, the only difference is that now two charges of opposite charge are frozen at the points  $\tau_1$  and  $\tau_2$  respectively, all others (whose number is not fixed) are free to move. We may write  $F(\tau_1 - \tau_2) \propto \exp\{-U(|\tau_1 - \tau_2|)\}$ , where  $U$  is the effective screened interaction between the two frozen opposite charges. In an analogous way  $\tilde{F}$  may be expressed through the screened interaction of two positive (negative) charges. It is important to stress that a single particle propagator (more generally: a correlator of an odd number of particles) *does not* have a gas equivalent. The reason is that different configurations of “charge” correspond to terms of different signs (an exchange of two charges gives rise to a sign change). Thus not all terms can be assigned a positive weight.

There are various ways to generalize and extend the model discussed here. First, we may include interactions in the lead and even between electrons in the lead and electrons on the dot. In an obvious notation we denote the two types of interactions by  $V_{ll}$  and  $V_{ol}$  respectively, while the interaction associated with the dot is  $V_{00}$ . We

assume that these (effective) interactions may be time dependent, but we do not include space dependence (only capacitive component). The only way these interactions modify the results is through the definition of the effective action, Eq. (8), where one substitutes

$$V(\omega) \rightarrow V_{00} + V_{ll} - 2V_{ol}. \quad (34)$$

To see why this is the case one should proceed exactly along the lines of Appendices A and B, introducing a doublet of Hubbard–Stratonovich fields,  $\phi_0, \phi_l$ .

Inclusion of the spin degree of freedom is trivial (redefinition of the DOS) as long as the tunneling does *not* conserve spin. If spin is conserved in the process of tunneling this is translated into a gas with charges (positive and negative) of two “colors”. Charges of the same color interact among themselves via the potential  $2 \ln \Omega|\tau| + S(\tau)$ , as in Eq. (15). Charges of different colors interact through the potential  $S(\tau)$ . This implies that in the absence of real interaction in the dot ( $S = 0$ ), the gas factorizes into two mutually non-interacting gases. Using an analogy with the multichannel Kondo problem one may claim that the separatrix line still exists in the



system's phase diagram. Its expression through the bare parameters of the dot requires, however, some more calculations.

## V. ACKNOWLEDGMENTS

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## APPENDIX A: SINGLE PARTICLE GREEN FUNCTION

Let us first calculate the imaginary time single particle Green function of an isolated dot described by a Hamiltonian Eq. (1). It may be written as [31]

$$\mathcal{G}_\alpha(\tau_i - \tau_f, \mu) = \frac{1}{Z(\mu)} \int \mathcal{D}[\bar{a}_\alpha(\tau) a_\alpha(\tau)] e^{-S[\bar{a}_\alpha, a_\alpha]} \bar{a}_\alpha(\tau_i) a_\alpha(\tau_f), \quad (\text{A1})$$

with the fermionic action given by

$$S[\bar{a}_\alpha, a_\alpha] = \int_0^\beta d\tau \left[ \sum_\alpha \bar{a}_\alpha(\tau) (\partial_\tau + \epsilon_\alpha - \mu) a_\alpha(\tau) + \frac{V^{[0]}}{2} \left[ \sum_\alpha \bar{a}_\alpha(\tau) a_\alpha(\tau) - \mathcal{N}_0 \right]^2 \right]; \quad (\text{A2})$$

here  $Z(\mu)$  is the partition function and  $\mu$  is the chemical potential. Splitting the interaction term in the action by means of the Hubbard–Stratonovich transformation with the auxiliary Bose field,  $\phi(\tau)$ , one obtains

$$\begin{aligned} \mathcal{G}_\alpha(\tau_i - \tau_f, \mu) &= \frac{1}{Z(\mu)} \int \mathcal{D}[\phi(\tau)] e^{-\int_0^\beta d\tau [\phi(\tau) [2V^{[0]}]^{-1} \phi(\tau) - i\mathcal{N}_0 \phi(\tau)]} \\ &\quad \int \mathcal{D}[\bar{a}_\alpha a_\alpha] e^{-\int_0^\beta d\tau \sum_\alpha \bar{a}_\alpha (\partial_\tau + \epsilon_\alpha - \mu + i\phi(\tau)) a_\alpha} \bar{a}_\alpha(\tau_i) a_\alpha(\tau_f) \\ &= \frac{1}{Z(\mu)} \int \mathcal{D}[\phi(\tau)] e^{-\int_0^\beta d\tau [\phi(\tau) [2V^{[0]}]^{-1} \phi(\tau) - i\mathcal{N}_0 \phi(\tau)]} Z^{[\phi]}(\mu) \mathcal{G}_\alpha^{[\phi]}(\tau_i, \tau_f, \mu) \end{aligned} \quad (\text{A3})$$

with the same transformations in  $Z(\mu)$ . Here  $Z^{[\phi]}(\mu)$  and  $\mathcal{G}_\alpha^{[\phi]}(\tau_i, \tau_f, \mu)$  are respectively the partition and Green functions of non-interacting electrons in the time dependent (but spatially uniform) potential,  $i\phi(\tau)$ . These quantities may be easily calculated, taking advantage of a gauge transformation. The result is

$$Z^{[\phi]}(\mu) = Z^{[0]}(\mu - i\phi_0), \quad (\text{A4})$$

$$\mathcal{G}_\alpha^{[\phi]}(\tau_i, \tau_f, \mu) = \mathcal{G}_\alpha^{[0]}(\tau_i - \tau_f, \mu - i\phi_0) e^{i \int_{\tau_i}^{\tau_f} d\tau [\phi(\tau) - \phi_0]}, \quad (\text{A5})$$

where  $Z^{[0]}(\mu) \equiv \exp\{-\beta\Omega^{[0]}(\mu)\}$  and  $\mathcal{G}_\alpha^{[0]}(\epsilon_n, \mu) = (i\epsilon_n - \epsilon_\alpha + \mu)^{-1}$  are the partition and Green functions of non-interacting electron gas; we have introduced Matsubara representation for the boson field,  $\phi(\tau)$ :

$$\phi_m \equiv \beta^{-1} \int_0^\beta d\tau \phi(\tau) \exp\{i\omega_m \tau\},$$

$\omega_m = 2\pi mT$ . For latter convenience we rewrite the exponent in Eq. (A5) in the following form

$$\exp \left\{ i \int_{\tau_i}^{\tau_f} d\tau [\phi(\tau) - \phi_0] \right\} = \exp \left\{ \beta \sum_{m \neq 0} \frac{\phi_{-m} J_m^{\tau_i|\tau_f}}{\omega_m} \right\}, \quad (\text{A6})$$

where  $J_m^{\tau_i|\tau_f}$  is the Matsubara transform of the following function

$$J_\tau^{\tau_i|\tau_f} = \delta(\tau - \tau_i) - \delta(\tau - \tau_f). \quad (\text{A7})$$

Transforming the functional integral over  $\phi(\tau)$  to integrals over the Matsubara components,  $\phi_m$ , we obtain

$$\mathcal{G}_\alpha(\tau_i - \tau_f) = \frac{1}{Z(\mu)} \int d\phi_0 e^{-\beta[\phi_0[2V^{[0]}]^{-1}\phi_0 - i\phi_0\mathcal{N}_0 + \Omega^{[0]}(\mu - i\phi_0)]} \mathcal{G}_\alpha^{[0]}(\tau_i - \tau_f, \mu - i\phi_0) \\ \int \prod_{m \neq 0} d\phi_m \exp \left\{ \beta \sum_{m \neq 0} \left[ -\frac{\phi_{-m}\phi_m}{2V^{[0]}} + \frac{\phi_{-m}J_m^{\tau_i|\tau_f}}{\omega_m} \right] \right\}. \quad (\text{A8})$$

For large enough systems ( $\Delta \ll T$ ) the integral over the static component,  $\phi_0$ , may be calculated in a saddle point approximation leading to  $\mathcal{G}_\alpha^{[0]}(\tau_i - \tau_f, \bar{\mu})$ , where the stationary point,  $\bar{\mu}$ , is the real solution of the equation  $(\mu - \bar{\mu})/V^{[0]} + N_0 + \partial\Omega^{[0]}(\bar{\mu})/\partial\mu = 0$ . The remaining integrals (over  $\phi_m$  for  $m \neq 0$ ) are purely Gaussian. As a result one obtains

$$\mathcal{G}_\alpha(\tau_i - \tau_f, \mu) = \mathcal{G}_\alpha^{[0]}(\tau_i - \tau_f, \bar{\mu}) e^{-S(\tau_i - \tau_f)}, \quad (\text{A9})$$

where

$$S(\tau) = T \sum_{m \neq 0} \frac{V^{[0]}}{\omega_m^2} (1 - e^{i\omega_m \tau}). \quad (\text{A10})$$

We shall include now the environment, Eq (5), in the calculations of the Green function. Assuming that the noise is Gaussian with zero mean value, one first perform the averaging over noise realizations as

$$\langle \dots \rangle_{noise} = \int \mathcal{D}[\eta(\tau)] e^{-\frac{1}{2} \int_0^\beta d\tau d\tau' \eta(\tau) K^{-1}(\tau - \tau') \eta(\tau')} \dots, \quad (\text{A11})$$

where  $K(\tau - \tau') = \langle \eta(\tau) \eta(\tau') \rangle$ . An important observation is that the partition function of the dot,  $Z(\mu)$ , is not affected by the noise term [32] (cf. Eq. (A4)). Averaging Eq. (A1) over the Gaussian noise, Eq. (A11), leads to an effective fermionic action with interaction which is non-local in time,

$$S_{int}[\bar{a}_\alpha, a_\alpha] = \frac{1}{2} \int_0^\beta \int_0^\beta d\tau d\tau' \sum_\alpha \bar{a}_\alpha a_\alpha \left( V^{[0]} \delta(\tau - \tau') - K(\tau - \tau') \right) \sum_\alpha \bar{a}_\alpha a_\alpha. \quad (\text{A12})$$

As a result, one obtains an effective renormalization (screening) of the zero-mode interaction potential

$$V^{[0]} \rightarrow V(\omega_m) = V^{[0]} - K(\omega_m). \quad (\text{A13})$$

Further calculations follow exactly the same steps as outlined above. The final result is given by Eqs. (7) and (8).

## APPENDIX B: MAPPING ONTO A COULOMB GAS

Our aim here is to derive an expression for the partition function corresponding to the full Hamiltonian, Eq. (6). Performing averaging over the noise and the Hubbard–Stratonovich transformation as described in Appendix A one obtains

$$Z = \int \mathcal{D}[\phi] \exp \left\{ - \sum_{m \neq 0} \frac{\phi_{-m}\phi_m}{2V(\omega_m)} \right\} Z[\phi] \quad (\text{B1})$$

where

$$Z[\phi] = \int \mathcal{D}[\bar{a}a\bar{d}d] \exp \left\{ - \int_0^\beta d\tau \mathcal{L} \right\} , \quad (\text{B2})$$

$$\mathcal{L} = \sum_\alpha \bar{a}_\alpha (\partial_\tau + \epsilon_\alpha + i\phi(\tau)) a_\alpha + \sum_k \bar{d}_k (\partial_\tau + \epsilon_k) d_k + W \sum_{k\alpha} \bar{d}_k a_\alpha + W^* \sum_{k\alpha} \bar{a}_\alpha d_k . \quad (\text{B3})$$

One may integrate out now all fermions of the dot and lead except the two living at the point of the tunneling contact by introducing the following resolutions of unity

$$1 = \int \mathcal{D}[s] \delta(s - \sum_\alpha a_\alpha) = \int \mathcal{D}[s, \bar{\sigma}] \exp \left\{ i \int_0^\beta d\tau \bar{\sigma} (s - \sum_\alpha a_\alpha) \right\} , \quad (\text{B4})$$

and similarly

$$1 = \int \mathcal{D}[r, \bar{\rho}] \exp \left\{ i \int_0^\beta d\tau \bar{\rho} (r - \sum_k d_k) \right\} , \quad (\text{B5})$$

where  $s, r, \bar{\sigma}, \bar{\rho}$  are Grassman variables. We also introduce the two corresponding identities for the conjugate variables. Performing the Grassman integration over the original variables  $\{a, \bar{a}, d, \bar{d}\}$  and then over the auxiliary  $\{\sigma, \bar{\sigma}, \rho, \bar{\rho}\}$  one obtains

$$Z[\phi] = Z^{[0]} Z^{[l]} \left\langle \exp \left\{ - \int_0^\beta d\tau (W \bar{r} s + W^* \bar{s} r) \right\} \right\rangle , \quad (\text{B6})$$

where the angular brackets denote integration over fields  $s$  and  $r$  with the following  $\phi$ -dependent measure:

$$\det \mathcal{G}^{[\phi]} \det \mathcal{G}^{[l]} \int \mathcal{D}[\bar{s} s \bar{r} r] \exp \left\{ - \int_0^\beta d\tau d\tau' [\bar{s} (\mathcal{G}^{[\phi]})^{-1} s + \bar{r} (\mathcal{G}^{[l]})^{-1} r] \right\} . \quad (\text{B7})$$

Here we have introduced notations for the traces of the Green functions of the dot and the lead respectively

$$\mathcal{G}^{[\phi]}(\tau, \tau') \equiv \sum_\alpha \mathcal{G}_\alpha^{[\phi]}(\tau, \tau') = \nu^{[0]} \frac{\exp \left\{ \beta \sum_{m \neq 0} \frac{\phi_{-m} J_m^{\tau \tau'}}{\omega_m} \right\}}{\tau - \tau'} , \quad (\text{B8})$$

$$\mathcal{G}^{[l]}(\tau - \tau') \equiv \sum_k \mathcal{G}_k^{[l]}(\tau - \tau') = \nu^{[l]} \frac{1}{\tau - \tau'} . \quad (\text{B9})$$

We have assumed that both the dot and the lead have continuous spectra, characterized by constant density of states ( $\nu^{[0]}$  and  $\nu^{[l]}$  respectively), and employed Eqs. (A5)–(A7). At finite temperatures one has to substitute

$$\frac{1}{\tau - \tau'} \rightarrow \frac{\pi/\beta}{\sin \pi(\tau - \tau')/\beta} . \quad (\text{B10})$$

The next step is to expand the exponent in Eq. (B6) to infinite order in  $W$ , yielding

$$\frac{Z[\phi]}{Z^{[0]} Z^{[l]}} = \sum_{M=0}^{\infty} \frac{|W|^{2M}}{(2M)!} \frac{(2M)!}{M! M!} \int_0^\beta \dots \int_0^\beta d\tau_1 \dots d\tau_{2M} \langle s_1 \bar{s}_2 \dots s_{2M-1} \bar{s}_{2M} \rangle \langle \bar{r}_1 r_2 \dots \bar{r}_{2M-1} r_{2M} \rangle , \quad (\text{B11})$$

where  $s_i \equiv s(\tau_i), i = 1, 2 \dots 2M$ . The combinational factor  $(2M)!/(M!M!)$  represents the number of possibilities to rearrange time sequences in the angular brackets. Due to the Gaussian nature of the measure, Eq (B7), one may apply Wick's theorem, resulting in the  $M \times M$  Slatter determinant construction

$$\langle \bar{r}_1 r_2 \dots \bar{r}_{2M-1} r_{2M} \rangle = \det |\mathcal{G}^{[l]}(\tau_{2n-1} - \tau_{2m})| ; \quad (\text{B12})$$

$n, m = 1, 2 \dots M$ . We employ now the explicit form of the Green function, Eq. (B9), along with the known properties of a Cauchy determinant [17] to write

$$\langle \bar{r}_1 r_2 \dots \bar{r}_{2M-1} r_{2M} \rangle = -(\nu^{[l]})^M \frac{\prod_{n < m} (\tau_{2n} - \tau_{2m}) \prod_{n < m} (\tau_{2n-1} - \tau_{2m-1})}{\prod_{n, m} (\tau_{2n} - \tau_{2m-1})} . \quad (\text{B13})$$

In the same manner

$$\begin{aligned} \langle s_1 \bar{s}_2 \dots s_{2M-1} \bar{s}_{2M} \rangle &= -(\nu^{[0]})^M \frac{\prod_{n < m} (\tau_{2n} - \tau_{2m}) \prod_{n < m} (\tau_{2n-1} - \tau_{2m-1})}{\prod_{n, m} (\tau_{2n} - \tau_{2m-1})} \\ &\exp \left\{ \beta \sum_{m \neq 0} \frac{\phi_{-m} J_m^{\tau_2 \dots \tau_{2M}} | \tau_1 \dots \tau_{2M-1}}{\omega_m} \right\} . \end{aligned} \quad (\text{B14})$$

where  $J_m$  is the Fourier transform of the imaginary time function

$$J_{\tau}^{\tau_2 \dots \tau_{2M}} | \tau_1 \dots \tau_{2M-1} = \sum_{j=1}^{2M} (-1)^j \delta(\tau - \tau_j) . \quad (\text{B15})$$

The exponent in Eq. (B14) is the only part of  $Z[\phi]$  which depends on the Hubbard–Stratonovich field,  $\phi(\tau)$ . Thus integration over the field  $\phi$ , Eq. (B1), may be easily performed for each term in the sum (Eq. (B11)) separately. Indeed

$$\int \mathcal{D}[\phi_m] \exp \left\{ -\beta \sum_{m \neq 0} \left[ \frac{\phi_{-m} \phi_m}{2V(\omega_m)} - \frac{\phi_{-m} J_m}{\omega_m} \right] \right\} = \exp \left\{ -\beta \sum_{m \neq 0} J_{-m} \frac{V(\omega_m)}{2\omega_m^2} J_m \right\} . \quad (\text{B16})$$

One may now return to time representation and employ the definition of  $J_{\tau}$ , Eq. (B15), as well as Eq. (8) to write the exponent on the r.h.s. of Eq. (B16) as  $\sum_{i < j}^{2M} (-1)^{i+j} S(\tau_i - \tau_j)$ . Note also that the product of the two Cauchy determinants (Eqs. (B13) and (B14)), being positive definite, may be written as an exponent of  $\sum_{i < j}^{2M} (-1)^{i+j} 2 \ln |\tau_i - \tau_j|$ . Finally one obtains Eq. (15) for the partition function of the dot connected to the lead.

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- $$V(q, \omega_m) = \left[ \frac{1}{V^{(0)}(q)} + \frac{dn}{d\mu} \frac{Dq^2 + \gamma_{in}}{Dq^2 + |\omega_m| + \gamma_{in}} \right]^{-1}.$$
- Here  $\frac{dn}{d\mu}$  is the thermodynamic density of states (compressibility),  $V^{(0)}(q)$  is the unscreened interaction. At  $q=0$  and  $\omega \gg \gamma_{in}$  this equation reduces to Eq. (10) with  $C_G = \infty$ ,  $V^{(0)}(q=0) = e^2/C$ , and  $R = \Delta/(2\gamma_{in})$  ( $\Delta$  being the mean level spacing).
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Table 1

	Quantum Dot	Coulomb Gas
(temperature) $^{-1}$	$\beta$	$2 + 2\tilde{r}$
linear size	L	$\beta$
dimension	"0" +1	1
potential	$V(\omega)$	$\ln(\Omega \tau )$
fugacity	$e^{\mu\beta}$	$\theta$